Study on PID Control Design and Electric Kettle Simulation

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Abstract

The simplicity, ease of implementation and robustness has attracted the use of Proportional, Integral and Derivative (PID) controllers. Numerous tuning techniques are available for tuning of PID controllers, each one of it has pros and cons. Majority of the tuning techniques are proposed for First Order System with Time Delay (FOPDT). This paper presents the technique for obtaining the FOPDT model and performance comparison of PID controller based on open loop, closed loop tuning techniques and PID controller tuned with Internal Model Control (IMC) technique for setpoint tracking and disturbance rejection.

Keywords: PID; Tuning methods; Open loop; Closed loop

1 Introduction

The Proportional Integral Derivative(PID) controller is the most popular control in industries for closed-loop control since it can promise satisfactory performances with a simple algorithm for many processes. It finds that about 98% of the regulatory controllers in the industry will use the PID algorithm. PID Control is commonly known as a three-term controller- the Proportional (P), Integral (I), and Derivative (D). An appropriate controller setting adjustment will help with better closed-loop system performance. And this procedure is known as controller tuning. Hundreds of methods, theories, and tools are available for tuning the PID controller. But still to find the optimal parameters for the PID controller is a tricky task in real life, and we still use the trial and error method for a tunning process by the control engineers.[9] The controller can provide optimized control action and minimized error performance with the optimum tuning of the three parameters in the PID controller algorithm[11]. The mathematical form of the PID algorithm is presented in equation (1).

$$G_{PID}(s) = G_C(s) = K_P \left(1 + \frac{1}{T_i s} + T_d s\right) \quad (1)$$

where $G_C(s)$ is the controller transfer function, K_P is the proportional gain, T_i is the integral time and T_d is the derivative time. This paper focuses on the various PID controller design, tuning methods as well as personal kettle simulation design.

2 Description of PID controller design

The step change of several simulated systems portrayed the planning procedure of the PID controller and compared the closed-loop response to show the capability of conventional strategies. The controller parameters are balanced by the changed strategy in the invigorated frameworks and changed to the closed-loop response of a similar system tuned by old-style PID tuning decisions utilized in modern applications.[5]

2.1 PID controller

The PID controller is a robust and simple system that provides control performance and is commonly known as the three-term controller. The ideal PID controller design with performance constraints is the most widely considered. Various manufacturers commonly use different algorithms essential for identifying these designs and whose differences from the ideal algorithms are used for tuning parameters. The parallel ideal PID design, non-interacting ideal PID design, and the interacting PID design are the most commonly used designs.[1] Figure (1) shows the standard block diagram of the PID controller.

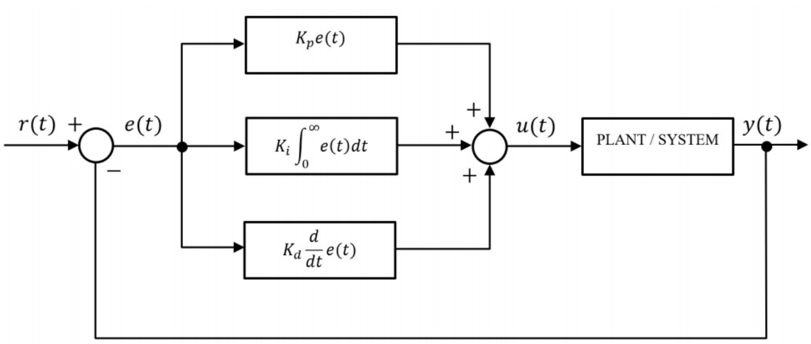

Figure 1: PID Controller Block Diagram

In the following equation (2), the PID controller is numerically represented.

$$u(t) = K_p \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right] \tag{2}$$

Here, the control and error signals of the system are represented by the proportion gain, the integral and derivative time constants respectively. In the following equation (3), the corresponding PID controller transfer function is given.

$$G_C(s) = K_P \left[1 + \frac{1}{T_i s} + T_d s \right] \tag{3}$$

Depending on the variable to be controlled, the PID tuning principles prescribe the configuration of the PID controller and also suggest the values to adjust the controller parameters. While the exact knowledge of the process dynamics is not typically required for the success of each tuning technique, the understanding of inherent characteristics of the individual components of the PID controllers is of particular importance. Equation (4) is the modified form of the above equation.

$$G_C(s) = K_P + \frac{K_I}{s} + K_D s \tag{4}$$

Here, the integral and derivative gain values of the controller are shown and the steady state error is not eliminated but the proportional controller will have the effect of reducing the rise time and will decrease. The integral controller will worsen the transient response but eliminates the sum of square error(SSE). The effect of increasing the robustness of the system, reducing the overshoot, and improving the transient response is achieved by the derivative controller. Importantly, the overall effect of these controllers greatly depends on each other. On the other hand, in time delay systems, PID controllers are generally used.

2.2 Time delay system

A novel low channel filter is proposed to determine the PID coefficient and update cancelation of the persistent and unstable first order shapes with time delay.[10] Furthermore, a first order lead-lag compensator is added to the PID controller to improve performance for the first order as well as second order unstable systems with time delay. Time-delay systems (TDS) are encountered in various fields, including engineering, science, and finance. In engineering, a time delay is a cause of instability and instabilities. Two types of time delay systems are there: frustrated and fair. In TDS, where time-delays remain present between the inputs to the system and their subsequent output, can be referred to as delaying difference conditions (DDEs).[2][3] Systems with delays belong to a class of infinite dimensional systems commonly encountered in the modeling and analysis of communication and diffusion phenomena. Time delays in control loops generally degrade system performance and make difficult the analysis and design of input controllers. Some TDS are researched and discussed in the following sub-section.

2.2.1 First-order plus time delay(FOPTD)

The model for FOPDT model is given as equation (5) below

$$P(s) = \frac{Ke^{-\tau s}}{Ts+1} \tag{5}$$

Here K is the gain, T is the time delay and τ is the time constant. To ensure the strength of the procedure, the time consistent ought to be picked wary, the nonexistent channel is gotten as equation (6) below

$$A(s) = \frac{1}{Bs+1} \tag{6}$$

For a wide scope of the proportion, we select the time steady. At the point we can consider as equation (7)

$$P(s) = \frac{Ke^{-\tau s}}{(Ts+1)(Bs+1)} \tag{7}$$

Consequently, the ideal feedback controller is given as equation (8)

$$C(s) = \frac{(\alpha s+1)(Ts+1)(Bs+1)}{(\lambda s+1)^2 - (\alpha s+1)e^{-\tau s}} = \frac{(\alpha s+1)(Ts+1)(Bs+1)}{K(2\lambda+\tau-\alpha)s(\frac{\lambda^2+\alpha\tau}{2\lambda+\tau-\alpha}s+1)} \tag{8}$$

Here $\frac{\lambda^2+\alpha\tau}{2\lambda+\tau+\alpha}s = T$ and $\alpha = \frac{2\lambda T+\tau T-\lambda^2}{\tau-T}$ for $\alpha > 0$. Since we know $\lambda < T + \sqrt{\tau T + T^2}$ and the tuning parameter is defined as $\lambda = B = 2\min(\tau, T)$. After simplification, writing internal model feedback controller into the structure of PID controller, we will get equation(9)

$$C(s) = \frac{(\alpha s+1)(Bs+1)}{K(2\lambda+\tau-\alpha)s} = \frac{\alpha+B}{K(2\lambda+\tau-\alpha)}\left(1+\frac{1}{(\alpha+B)s}+\frac{\alpha B}{(\alpha+B)}s\right) \tag{9}$$

2.2.2 Second-order plus time delay (SOPID)

Assume that $T \geq L$. Consider a second-order plus time delay system is given as equation (10)

$$P(s) = \frac{Ke^{-\tau s}}{(Ts+1)(Ls+1)} \quad (T \geq L) \tag{10}$$

The ideal feedback controller is given as equation (11)

$$C(s) = \frac{(\alpha s+1)(Ts+1)(Ls+1)}{(\lambda s+1)^2 - (\alpha s+1)e^{-\tau s}} \tag{11}$$

The formulas to calculate λ depend on time day / time constant ratio of the process. And for a wide range, the tuning parameter is defined as $\lambda = \sqrt{\tau L + L^2}$.

2.2.3 Integerating process plus time delay (IPTD)

Consider an integrating process plus time system as follows equation(12)

$$P(S) = \frac{Ke^{-\tau s}}{s} \tag{12}$$

Assume the integrating plus time delay process series an imaginary first-order filter, which does not exist in the close-loop system. And the imaginary filter is obtained as equation(13)

$$A(s) = \frac{1}{Bs+1}B = 0.5\tau \tag{13}$$

Then we can consider the time delay process is given as equation(14)

$$P(s) = \frac{Ke^{-\tau s}}{s(Bs+1)} \tag{14}$$

In the below, the integrating process is approximated by a first order process,

$$\frac{K}{s} \approx \frac{K}{s+1} \tag{15}$$

Here, is a large value constant. Then, the ideal feedback controller is presented as equation(16)

$$C(s) = \frac{(s+1)(\alpha s+1)(Bs+1)}{K(2\lambda+\tau-\alpha)s(\frac{\lambda^2+\alpha\tau}{2\lambda+\tau-\alpha}s+1)} \tag{16}$$

where $\frac{\lambda^2+\alpha\tau}{2\lambda+\tau-\alpha} =$, and we can get $\alpha = \frac{2\lambda+\tau-\lambda^2}{\tau+}$. The tuning parameter is defined as $\lambda = 0.5T + \sqrt{\tau T + T^2}$, where $T = B$

2.2.4 First-order delayed integrating process (FOPID)

Consider the following first-order delayed integrating process process is given as equation(17)

$$P(s) = \frac{Ke^{-\tau s}}{s(Ts+1)} \tag{17}$$

THe ideal feedback controller is same as IPTD. We series a lead-lag compensator with teh PID controller to gurantee the stability and improve the performance of the process, and the compensator is given as equation(18)

$$P(s) = \frac{0.5\tau s+1}{0.01s+1} \tag{18}$$

Then the PID controller in series with a lead-lag compensator as follows equation(19)

$$C(s) = \frac{\alpha+T}{K(2\lambda+\tau-\alpha)}(1+\frac{1}{\alpha+Ts}+\frac{}{\alpha+T}s)\frac{0.5\tau s+1}{0.01s+1} \tag{19}$$

And the tuning parameter is defined as $\lambda = T + \sqrt{\tau T + T^2}$

3 Tuning

Preselection of a controller structure can pose a challenge in applying PID control. As vendors often recommend their own designs of controller structures, their tuning rules for a specific controller structure do not necessarily perform well with other structures. Nevertheless, controller parameters are tuned such that the closed-loop control system would be stable and would meet given objectives associated with the following:

- Stability and robustness
- Set-point following and tracking performance at transient, including rise-time, overshoot, and settling time
- Regulation performance at steady-state, including load disturbance rejection
- Robustness against plant modeling uncertainty

- Noise attenuation and robustness against environmental uncertainty

With the given objectives, tuning methods for PID controllers can be grouped according to their nature and usage, as follows:

- **Analytical method** - PID parameters are claculated from analytical or algebraic relations between a plant model and an objective (such as internal model control or lambda tuning). These can lead to an easy-to-use formula and can be suitable for use with online tunning, but the objective needs to be in an analytical form and the model must be accurate.
- **Heuristic methods** - These are evolved from practical experience in manual tuning (such as Z-N tuning rule) and from artificial intelligence (including expert systems, fuzzy logic, and neural networks). Again, these can serve in the form of a formula or a rule base for online use, often with tradeoff design objectives.
- **Frequency response methods** - Frequency characteristics of the controlled process are used to tune the PID controller (such as loop-shaping). These are often offline and academic methods, where the main concern of design is stability and robustness.[4]
- **Optimization methods** - These can be regarded as a special type of optimal control, where PID parameters are obtained ad hoc using an offline numerical optimization method for a single composite objective or using computerized heuristics or an evolutionary algorithm for multiple design objectives. These are often time-domain methods and mostly applied offline.
- **Adaptive tuning methods** - These are for automated online tuning, using one or a combination of the previous methods based on real-time identification.

The previous classification does not set an artificial boundary and some methods applied in practice may belong to more than one category. However, no tuning method so far can replace the simple Z-N method in terms of familiarity and ease of use to start with. Furthermore, there is a lack of methods that are generic and can be quickly applied to the design of onboard or on-chip controllers for a wide range of consumer electronics, domestic appliances, mechatronic systems, and microelectromechanical systems. Over the past half-century, search goes on to find the next key technology for PID tuning and modular realization.[6][7]

3.1 The ZN step response method

The system is also automated by the use of the step response tuning technique. At fixed set-point and step size are used to determine the PID controller parameters. The PID settings result in a large overshoot and an oscillatory response. The result is based on simulation of infinite. The key criterion is a quarter decay ratios. Various researchers have tackled the procedure based on experimental values and proposed various settings to improve the technique. Ziegler and Nichols proposed a tuning strategy for PID controller which is commonly used in industries. This is a manual mode based on an open-loop step response of a system. It is proposed for a first order system with dead time plant model as shown in equation(20).

$$G(s) = \frac{k}{1+sT}e^{-sL} \tag{20}$$

where, L is the delay time, T is the time constant and k stands for static gain of the controller. In Figure 2, the output response of the ZN tuning method is illustrated.

The ZN tuning principles rely on what is known as a definitive sensitivity method. It consists of determining the point where the Nyquist plot of the open-loop system intersects the negative real axis. This point is obtained by attaching a purely proportional controller to the system, and by increasing the controller gain until the closed-loop system reaches the limit, at which oscillation occurs. The oscillation period is signified and the corresponding critical gain by the ZN rule for the three PID parameters is given as equation.

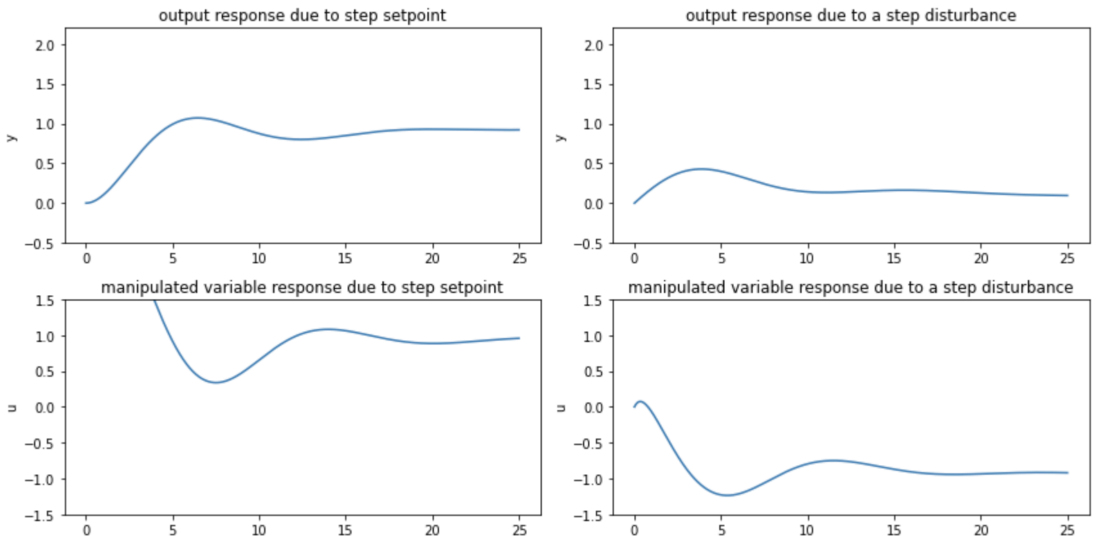

Figure 2: ZN Algorithm Simulation

3.2 The CC method for tuning PID controller

The CC tuning method for PID controller is an extension to the ZN method. ZN method shows a slow steady-state response. However, the CC method for tuning can overcome this limitation. It uses PID parameters obtained from open-loop frequency response. It gives improved result over ZN method if there is a significant process delay compared to the open-loop time constant. The system's response is shown to a phase change using the CC method. The system response is not only affected by the dynamics of the process but also by the characteristics of the measuring sensor and limited control element. There are several ways to determine the characteristics to be used for the proportional, integral, and derivative parameters in the controller, and the CC method is one of the methods. By observing the system's response to manual step changes without the controller working, initial characteristics for the PID parameters and then tune them manually are selected. The system's response is shown to a phase change as a first order response with dead time, using the CC method. From this response, three parameters and the output steady-state divided by the input step change, is the time constant of the first order response, and is the dead time. The system response is calculated as equation(21)

$$G_{PRC}(s) = \frac{Y_M(s)}{C(s)} = \frac{Ke^{-tTs_d}}{\tau s + 1} \tag{21}$$

CC method used the approximated mode and estimated the value of the parameters K, τ and t_d as indicated above.

3.3 Genetic algorithm for PID parameters tuning

GA is a heuristic optimization algorithm, which is evolved through the process of natural selection. GA is initialized by a starting population that includes a number of chromosomes and each represents a solution to the problem. According to the fitness function, the performance is evaluated in GA. Based on the fitness of each individual and certain probability, a selection of chromosomes is chosen to undergo three general stages. The stages are selection, crossover, and mutation. The purpose of these three main operations is to enable the evolution of new individuals to find the best solution. The implementation of genetic algorithm in PID tuning is given as below:

Process1: Initial setting of GA parameters

GA is executed by the initial population size. This parameter is important to allow the controller to be optimized as fast as possible. In this study, the number of individuals is set as 20, crossover rate, mutation rate, and the number of generations. The initial population is generated by encoding the PID parameters, and into binary representation known as chromosome. The binary encoding scheme is given by equation(22)

$$2^{m_j - 1} < b_j - a_j \times 10^4 \leq 2^{m_j} - 1 \tag{22}$$

Process2: Evaluate the fitness of each chromosome

The fitness of each chromosome is evaluated by converting its binary representation into real values which are represented as PID parameter values and substitute into its prediction model (also known as fitness function). The conversion of each chromosome into real values is made through decoding from binary representation can be expressed in equation(23).

$$x_j = a_j + \text{decimal}(\text{substring}_j) \times \frac{(b_j - a_j)}{2^{m_j} - 1} \tag{23}$$

In order to evaluate an initial fitness value, each set of PID parameters is substituted into the objective function. In this study, the MSE and IAE are preferred as objective function which is obtained from PID control system. The goal of GA is to search for minimum fitness value.

Process3: Selection and reproduction using a probabilistic method

Now, the whole fitness values and its corresponding chromosome will undergo selection operation. The higher probability of an individual in the population should be preferred for superior fitness value. Due to the feasible computation, tournament selection is preferred for improved selection scheme such that it is capable to handle the diversity power and population diversity to produce GA input generation. Roulette selection allows the poor chromosomes to be selected multiple times and also prevent large population size.

Process4: Implement crossover operation on the reproduced chromosomes

The crossover will be performed after the selection operation is completed. For basic GA, single point crossover is selected. The two parent chromosomes are randomly pick one cut-point and replace the exact part of the two parents to create offspring. The flowchart of a GA algorithm is shown in Figure (3).

Process5: Execute mutation operation with low probability

Mutation is used to prevent the algorithm to be stuck in local minima and maintain diversity in the population. Typically, low mutation rate should be preferred. Higher mutation rate may likely causes the input generation will turn into random search.

Process6: Repeat step 2 until the stopping criterion is met

After the selection, crossover, and mutation operation, again its binary sequence in each chromosome in the population needs to be decoded into real values in the subsequent generation. A new set of PID parameters is substituted into the PID control system to evaluate for new fitness value. This process will repeat over the course of steps 3, 4 and 5 successively until the end of generation where the optimal fitness is achieved.[8]

Hereditary calculation has been executed for alteration of PID controller for combination electric vehicle. A while later it differentiate the introduction of combination electric vehicle among adjustment of PID controller unsurprising change system, for example, ZN process and GA and it is set up that hereditary calculation is best amidst further technique.

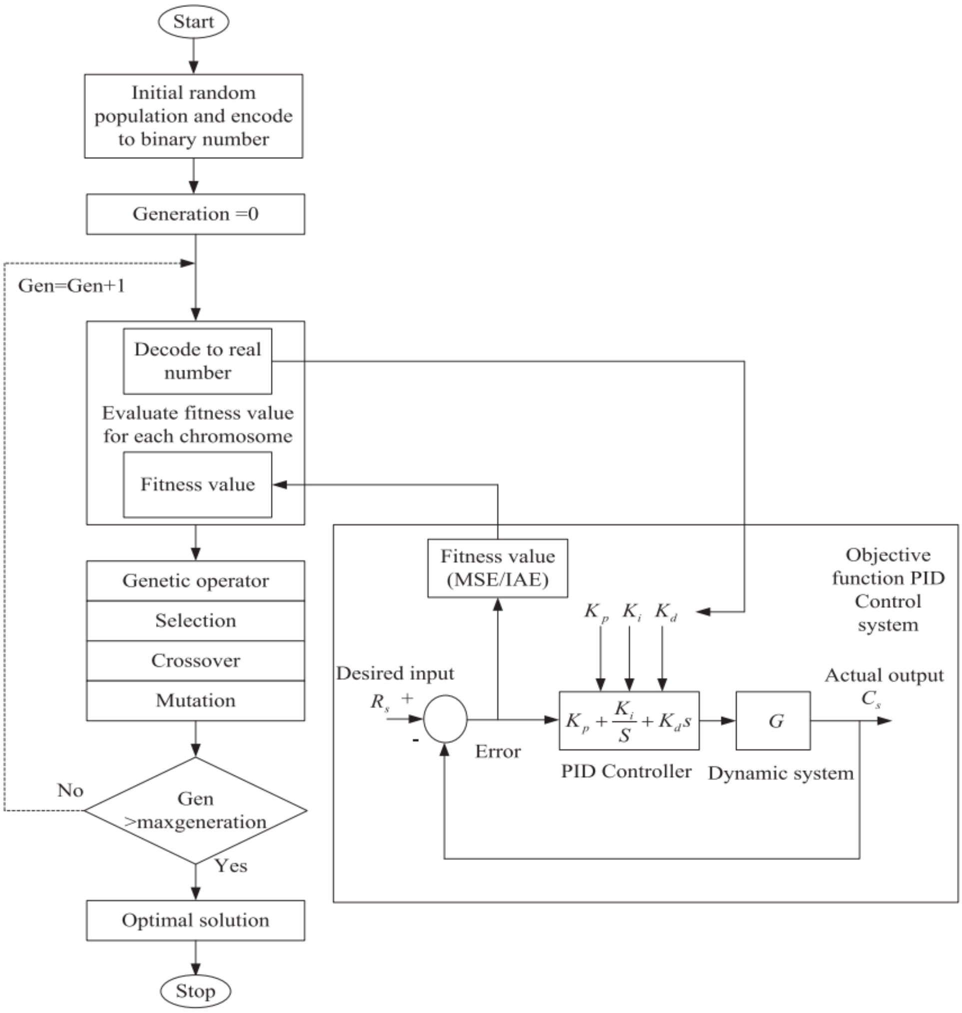

Figure 3: GA Algorithm Flowchart

4 Kettle Simulation

A typical electric kettle consists of two electrical components, which are heat element and on-off switch. When electric power is supplied to the heat element, it will heat up the water inside the kettle at full power to bring the water to higher temperature. Based on this simple process, an electric kettle serves the purpose of controlling the temperature of the water.

In this simulation, we use PID controller to help with controlling loop feedback mechanism. PID control signal is the sum of three terms which are based on error measurement, that are

$$u(t) = K_p + K_i \int_0^{\inf} e(t)dt + K_d \frac{d}{dt} e(t) \tag{24}$$

where $e(t) = r(t) - y(t)$. K_p is proportional constant, K_i is integral constant and K_d is derivative constant. The value of K_p, K_i and K_d are the key in providing stable and desired transient response which can be obtained by using heuristic methods.

The procedure of determining the PID gains is explained as follows:

- Set all gains to zero
- Increase the K_p gain until the response steadily oscillate.
- Increase the K_d gain until the oscillations significantly reduced.
- Repeat step(b) and step(c) until the response is stable with minimal oscillation.
- Increase the K_i gain to bring the response to the set point.

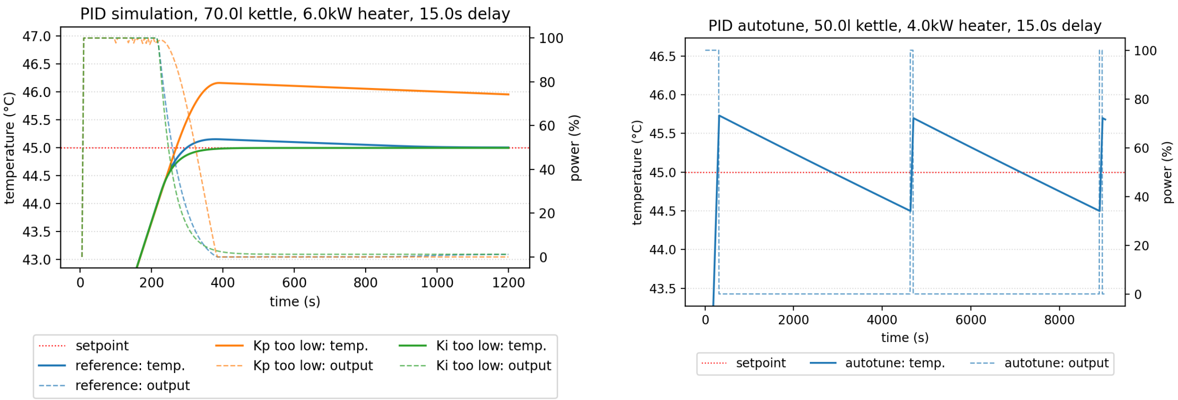

Figure 4: PID Comparison (Left) and Autotune Simulatin (Right)

5 Conclusion

PID, a structurally simple and generally applicable control technique, stems it success largely from the fact that it just works very well with a simple and easy to understand structure. In this work, PID tuning methods are analyzed for plants with transfer functions of the following types: first-order system, second-order system. In addition, the setting time assignment with input constraint is also analysed. Kettle simulation is used to illustrate the proposed method and validate the feasibility of my PID design.

References

[1] Hagglund T. Astrom KJ. *The future of PID control.* Control Eng Pract. 2001.

[2] Yue SL. Chi-Tsung H. *Tuning PID controller for open-loop unstable processes with time delay.* Chem Eng Commun. 1995.

[3] Liu GP. Daley S. *Optimal PID tuning using direct search algorithms.* Comput Control Eng J. 1999.

[4] Ksouri M. Farkh R, Laabidi K. *PI control for second order delay system with tuning parameter optimization.* Int J Electr Electron Eng. 2009.

[5] Norbert H. *All stabilizing PID controllers for time delay systems.* Automatica. 2009.

[6] Krstic M. Killingsworth NJ. *PID tuning using extremum seeking: online, model-free performance optimization.* IEEE Control SystMag. 2006.

[7] Budman H. Madhuranthakam CR, Elkamel A. *Optimal tuning of PID controllers for FOPTD. SOPTD and SOPTD with lead processes.* Chem Eng Process. 2008.

[8] Chidambaram M. Padma Sree R, Srinivas MN. *A simple method of tuning PID controllers for stable and unstable FOPTD systems.* Comput Chem Eng. 2004.

[9] Ho-Wang F Qiang B Yu Z. Qing-Guo W, Tong-Heng L. *PID tuning for improved performance.* IEEE Trans Contr Syst Technol. 1999.

[10] Chan CT. Wang FS, Juang WS. *Optimal tuning of PID controllers for single and cascade control loops.* Chem Eng Commun. 1995.

[11] Gaing ZL. *A particle swarm optimization approach for optimum design of PID controller in AVR system*. IEEE Trans Energy Convers 2004. 2004.